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Generating Functions for Measures of Inaccuracy

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Abstract:- In the present year we have obtained generating functions for several measure of inaccuracy. Our results include several well-known results.

Key words:- Generating functions, measure of inaccuracy, probability distributions, measure of entropy

1. Introduction:

Let $P = (p_1, p_2, \ldots \ldots p_n), Q = (q_1, q_2, \ldots \ldots q_n)$ (1)

be two probability distributions, then Kapur [1994, p. 163] defined measures of inaccuracy generating function.

$$h(t) = \sum_{i=1}^{n} p_i\, q_i \quad (2)$$

with the property

$$h^1(0) = \sum_{i=1}^{n} p_i\, ln q_i \quad (3)$$

which is Kerridge's [1961] measure of inaccuracy. This measure of inaccuracy is based on Shannon's [1948] measure of entropy. Renyi [1961] has generalized Shannon's measure of entropy by introducing a parameter

$\alpha(\alpha > 0, \alpha \neq 1)$

as $R_\alpha(P) = \frac{ln \sum_{i=1}^{n} p_i{}^\alpha}{1-\alpha}$ (4)

$\text{Lt}_{\alpha\to 1}\, R_\alpha(P) = \sum_{i=1}^{n} p_i\, ln p_i = S(P)$ Shannon's measure of entropy.

Kapur [1994 p.163] has generalized $h(t)$ i.e. generating function for Kerridge's [1961] measure of inaccuracy with the help of parameter α as

$$h_\alpha(t) = \frac{1}{1-\alpha}\left[\frac{\sum_{i=1}^{n} p_i{}^\alpha}{\sum_{i=1}^{n} p_i{}^\alpha q_i{}^{1-\alpha}} - 1\right], \alpha \neq 1 \alpha > 0 \quad (4^*)$$

$$h^1{}_\alpha(0) = \frac{1}{1-\alpha} ln\left[\frac{\sum_{i=1}^{n} p_i{}^\alpha}{\sum_{i=1}^{n} p_i{}^\alpha q_i{}^{1-\alpha}}\right], \alpha \neq 1 \alpha > 0 \quad (5)$$

which is Renyi's measure of inaccuracy, so $h_\alpha(t)$ can be regarded as a generating functions for Renyi's measure of inaccuracy.

In section 2 of present paper we have obtained generating function for measure of inaccuracy corresponding to different measure of entropy.

2(I) Let

$$h_{\alpha,\beta}{}^{(t)} = \frac{1}{\beta-\alpha}\left[\left\{\left(\frac{\sum_{i=1}^{n} p_i{}^\alpha}{\sum_{i=1}^{n} p_i{}^\alpha q_i{}^{1-\alpha}}\right)\left(\frac{\sum_{i=1}^{n} p_i{}^{1-\beta}}{\sum_{i=1}^{n} p_i{}^\beta}\right)\right\}^t - 1\right] \quad (6)$$

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where $0 < \alpha < 1, \beta > 1$ or $0 < \beta < 1, \alpha > 1$

$$h^1{}_{\alpha,\beta}{}^{(0)} = \frac{1}{\beta-\alpha} ln\left[\left(\frac{\sum_{i=1}^n p_i{}^\alpha}{\sum_{i=1}^n p_i{}^\alpha q_i{}^{1-\alpha}}\right)\left(\frac{\sum_{i=1}^n p_i{}^{1-\beta}}{\sum_{i=1}^n p_i{}^\beta}\right)\right] \quad (7)$$

which is a measure of inaccuracy corresponding to Kapur [1994, p.111] measure of entropy

$$\frac{1}{\beta-\alpha} ln\left(\frac{\sum_{i=1}^n p_i{}^\alpha}{\sum_{i=1}^n p_i{}^\beta}\right) \quad (8)$$

So, $h_{\alpha,\beta}{}^{(t)}$ is generating function for Kapur's [1994, p. 111] measure of inaccuracy. It is apparent that $h_{\alpha,1}{}^{(t)} = h_\alpha(t)$ and

$$\underset{\alpha\to 1}{\text{Lt}}\, h_{\alpha,1}{}^{(t)} = h(t)$$

(II)

$$h^k{}_{\alpha,\beta}(t) = \frac{1}{\beta-\alpha}\left[\left\{\left(\frac{\sum_{i=1}^n p_i{}^\alpha}{\sum_{i=1}^n p_i{}^\alpha q_i{}^{1-\alpha}}\right)^t \left(\frac{\sum_{i=1}^n p_i{}^\beta q_i{}^{1-\beta}}{\sum_{i=1}^n p_i{}^\beta}\right)\right\}^{kt} - 1\right] \quad (9)$$

$k > 0, 0 < \alpha < 1, \beta > 1$ or

$k > 0, 0 < \beta < 1, \alpha > 1$

$$[h^k{}_{\alpha,\beta}(0)]$$

$$= \frac{1}{\beta-\alpha}\left[\ln\left\{\left(\frac{\sum_{i=1}^n p_i{}^\alpha}{\sum_{i=1}^n p_i{}^\alpha q_i{}^{1-\alpha}}\right)\left(\frac{\sum_{i=1}^n p_i{}^\beta q_i{}^{1-\beta}}{\sum_{i=1}^n p_i{}^\beta}\right)\right\}^k\right] (10)$$

Which is measure of inaccuracy corresponding to Kapur's [1994. P. 113] measure of entropy

$$\frac{1}{\beta-\alpha} ln \frac{\sum_{i-1}^n p_i{}^\alpha}{(\sum_{i-1}^n p_i{}^\beta)^k}$$

So $h^k{}_{\alpha,\beta}(t)$ is generating function for Kapur's [1994, p. 113] measure of inaccuracy. It is obvious that

$$h^1{}_{\alpha,\beta}(t) = h^k{}_{\alpha,\beta}(t) h^1{}_{\alpha,1}(t) = h_\alpha(t)$$

$$\underset{\alpha\to 1}{\text{LT}}\, h_\alpha(t) = h(t)$$

Thus $h^k{}_{\alpha,\beta}(t)$ includes all the generating functions stated above.

(III)

Let $h(t) = \sum_{i=1}^n p_i\, q_i{}^t - \sum_{i=1}^n (1-p_i)(1-q_i)^t$ (11)

$$h^1(0) = \sum_{i=1}^n p_i\, lnq_i - \sum_{i=1}^n (1-p_i) ln(1-q_i) \quad (12)$$

which is measure of inaccuracy and corresponding to Fermi Dirac [c.f. Kapur 1994, p. 102] measure of entropy.

$$-\sum_{i=1}^n p_i\, lnp_i - \sum_{i=1}^n (1-p_i)\ln(1-p_i) \quad (13)$$

So $h(t)$ is generating function for Fermi Dirac's Kapur [1994, p. 102] measure of inaccuracy

(IV) Let $h(t) = \sum_{i=1}^n (p_i - a_i)(q_i - a_i)^t$-$\sum_{i=1}^n (b_i - p_i)(b_i - q_i)^t + \sum_{i=1}^n (b_i - a_i)^t$ (14)

then$h'(0) = \sum_{i=1}^n (p_i - a_i) ln(q_i - a_i) - \sum_{i=1}^n (b_i - p_i) ln(b_i - q_i) + \sum_{i=1}^n (b_i - a_i) ln(b_i - a_i)$ ... (15)

Which is a measure of inaccuracy corresponding to Kapur's [1994, p. 322] measure of entropy.

$$-\sum_{i=1}^n (p_i - a_i) ln(p_i - a_i) - \sum_{i=1}^n (b_i - p_i) ln(b_i - p_i) + \sum_{i=1}^n (b_i - a_i) ln(b_i - a_i) \ldots \quad (16)$$

$$a_i < p_i < b_i \quad (17)$$

so$h(t)$ is generating function for Kapur's [1994, p. 322] measure of inaccuracy.

If we put $a_i = 0$, $b_i = 1$ which is a natural constraint in (14)(15)(16)(17),(14) is reduced to (11), (16) is reduced to (13)

(V)

Let $h(t) = -\sum_{i=1}^n (p_i - a_i)(q_i - a_i)^t - \sum_{i=1}^n (b_i - p_i)(b_i - q_i)^t + \frac{1}{a}\sum_{i=1}^n \{1 + a(p_i - a_i)\}\{1 + a(q_i - a_i)\}^t +$

$\frac{1}{a}\sum_{i=1}^{n}\{1+a\,(b_i-p_i)\}\{1+a(b_i-q_i)\}^t -$

$\sum_{i=1}^{n}\left\{\left(\frac{b_i-a_i}{a}\right)(1+a)^{t+1}\right\}$ (18)

then

$$h^{1}(0) = -\sum_{i=1}^{n}(p_i-a_i)ln(q_i-a_i) - \sum_{i=1}^{n}(b_i - p_i)ln(b_i-q_i)$$

$$+\frac{1}{a}\sum_{i=1}^{n}\{1+a\,(p_i-a_i)\}\ln\{1+a(q_i-a_i)\} + \frac{1}{a}\sum_{i=1}^{n}\{1+a\,(b_i - p_i)\}\ln\{1+a(b_i-q_i)\}$$

$$-\sum_{i=1}^{n}\left\{\left(\frac{b_i-a_i}{a}\right)(1+a)\ln(1+a\right\} \quad (19)$$

which is a measure of inaccuracy corresponding to Kapur's [1994, p. 328] measure of entropy.

$$-\sum_{i=1}^{n}(p_i-a_i)ln(p_i-a_i) + \frac{1}{a}\left[\sum_{i=1}^{n}(p_i-a_i)ln\{1+a(p_i-a_i)\}\right]$$

$$-\sum_{i=1}^{n}\{(b_i-p_i)\}\ln\{(b_i-p_i)\} + \frac{1}{a}\sum_{i=1}^{n}\{1+a\,(b_i - p_i)\}\ln\{1+a(b_i-p_i)\}$$

$$-\sum_{i=1}^{n}\left\{\left(\frac{b_i-a_i}{a}\right)(1+a)\ln(1+a\right\} \quad (20)$$

so, $h(t)$ is generating function for Kapur's [1994, p.328] measure of inaccuracy

(VI) Let $h(t) = -\sum_{i-1}^{n} p_i\, q_i{}^t + \sum_{i=1}^{n}(1+p_i)\,(1+q_i)^t - 2^{t+1}$ (21)

then$h^{1}(0) = \sum_{i=1}^{n} p_i\, lnq_i + \sum_{i=1}^{n}(1+p_i)\ln(1+q_i) - 2ln2$ (22)

which is a measure of inaccuracy corresponding to Bose Einstein's [c.f. 1994 Kapur p. 102] measure of entropy

$$-\sum_{i=1}^{n} p_i\, lnp_i + \sum_{i=1}^{n}(1+p_i)\ln(1+p_i) - 2ln2 \quad (23)$$

so$h(t)$ is generating function for Bose Einstein's measure of inaccuracy.

(VII) Let

$$h_{\alpha,\beta}{}^{(t)} = \frac{1}{1-\alpha}\left[\left\{\left(\frac{\sum_{i=1}^{n} p_i{}^{\alpha+\beta-1}}{\sum_{i=1}^{n} p_i{}^{\alpha+\beta-1} q_i{}^{2-\alpha-\beta}}\right)\left(\frac{\sum_{i=1}^{n} p_i{}^{\beta}\, q_i{}^{1-\beta}}{\sum_{i=1}^{n} p_i{}^{\beta}}\right)\right\}^t - 1\right] \quad (24)$$

where α >0, β >0 α + β -1>0, α≠1

$$h^{1}{}_{\alpha,\beta}{}^{(0)} = \frac{1}{1-\alpha}\, ln\left\{\left(\frac{\sum_{i=1}^{n} p_i{}^{\alpha+\beta-1}}{\sum_{i=1}^{n} p_i{}^{\alpha+\beta-1} q_i{}^{2-\alpha-\beta}}\right)\left(\frac{\sum_{i=1}^{n} p_i{}^{\beta}\, q_i{}^{1-\beta}}{\sum_{i=1}^{n} p_i{}^{\beta}}\right)\right\} \quad (25)$$

which is a measure of Inaccuracy corresponding to Kapur's [1991, p. 6] measure of entropy so

$h_{\alpha,\beta}{}^{(t)}$is generating function for Kapur's [1991, p. 6] measure of inaccuracy.

It is apparent that when $\beta = 1 h_{\alpha,\beta}{}^{(t)}$ is reduced to $h_\alpha(t)$ given by (4*) and if $\alpha = 0$ in addition to $\beta = 1$

$h_{0,1}{}^{(t)} = n^{t-1}$

so

$h^{1}{}_{0,1}{}^{(0)} = \ln n$

which is measure of inaccuracy corresponding to Hartley's measure of entropy.

Thus $h_{0,1}{}^{(t)}$ is generating function for Hartley's measure of inaccuracy.

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References

[1] Kapur J.N. (1993) : “Maximum Entropy Models in Science and Engineering” Wiley Eastorn, New Delhi

[2] Kapur J.N. (1994) : “Measure of Information and their application” New Age Internationsl Limited, New Delhi

[3] Kerridge D.F. (1961) : “Inaccuracy and Inference” J. Royal Statist, Soc. Ser B,23, 184-194

[4] Renyi A. (1961) : “On measures of entropy and information” Proc. 4thBerkSymp Statist Problem 546-561

[5] Shannon C.E. (1948) : “A Mathematical Theory of communication” Bell System Tech. Journal Vol. 27, pp. 379-423, 623-659

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